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Charmonium Production in High Energy Collisions¹

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1. INTRODUCTION

One motivation for studying the production of charmonium in high energy collisions is that these particles are among the simplest probes for heavy-ion collisions. In recent years, there have been significant developments in heavy quarkonium production, both in theory and in experiment. These developments may have important implications for the use of charmonium as a probe in heavy-ion collisions. A summary of these developments is presented below. Those interested in more details are referred to a recent review article by Braaten, Fleming, and Yuan [1].

2. CHARMONIUM AS A PROBE

The simplest probes for hard processes in heavy-ion collisions are leptons and photons. One can argue that the next simplest are the heavy quarkonium states, bottomonium and charmonium. The bottomonium states are too heavy to be produced in abundance in heavy-ion collisions, but charmonium states are produced reasonably copiously. The states with the cleanest experimental signatures are the $J^{PC} = 1^{--}$ states J/ψ and ψ' , which decay into e^+e^- and $\mu^+\mu^-$. The J^{++} states χ_{cJ} , J=1,2, also have reasonably good experimental signatures through their radiative decays into $\psi + \gamma$.

The reason that these charmonium states are such simple probes is that they are essentially 2-body systems. The ψ , for example, is to a good approximation a bound state of a $c\bar{c}$ pair in a color-singlet state with angular-momentum quantum numbers 3S_1 . The color wavefunction is $(R\bar{R}+B\bar{B}+G\bar{G})/\sqrt{3}$, the space wavefunction has the form $\psi(\mathbf{r})=R(r)/\sqrt{4\pi}$, where R(r) is the radial wavefunction, and the possible spin wavefunctions are $\uparrow\uparrow$, $(\uparrow\downarrow+\downarrow\uparrow)/\sqrt{2}$, and $\downarrow\downarrow$. This $c\bar{c}$ state is the dominant component of the wavefunction, but there are also higher Fock state components. There is, for example, a small $c\bar{c}g$ component. Since the overall color state of the $c\bar{c}g$ must be color-singlet, the $c\bar{c}$ pair must be in a color-octet state. The 8 independent color-octet states are $R\bar{G}$, $G\bar{B}$, $B\bar{R}$, $B\bar{B}$, $B\bar{G}$, $G\bar{R}$, $(R\bar{R}-G\bar{G})/\sqrt{2}$, and $(G\bar{G}-B\bar{B})/\sqrt{2}$. For most observables, the contribution from the $c\bar{c}g$ component of the wavefunction is small. However, for some observables, the contribution from the $c\bar{c}$ component of the wavefunction is suppressed and components in which the $c\bar{c}$ pair are in a color-octet state can be important.

There are three important features of charmonium that make it simpler than light hadrons. For each of these simplifications, there is a theoretical tool that can be applied

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to charmonium that is not available in the case of light hadrons. These tools give us a handle on the effects of gluons in each of the three most important ranges of wavelength. These three ranges of wavelengths are much less than, comparable to, and much greater than r, where r is the typical separation of the $c\bar{c}$ pair in charmonium. The first simplifying feature of charmonium is that the mass of the charm quark $(m_c \approx 1.5 \text{ GeV})$ is much larger than the momentum scale Λ_{QCD} associated with nonperturbative effects in QCD. This allows us to use perturbation theory in the running coupling constant $\alpha_s(m_c)$ to calculate the effects of gluons whose wavelengths are of order $1/m_c$ or larger. The second simplifying feature is that the c and \bar{c} are nonrelativistic: the typical value of the relative velocity v of the c and \bar{c} is about $\frac{1}{2}$, so $v^2 \approx \frac{1}{4}$. This allows us to use a relativistic expansion in powers of v^2 to organize the effects of gluons whose wavelengths are comparable to r, which is on the order of $1/(m_c v)$. The third simplifying feature is that the geometrical size of charmonium states is significantly smaller than that of light hadrons: the typical separation r of the c and \bar{c} is only about 0.2 fm. This enables us to use the multipole expansion in powers of r/λ to simplify the interactions of gluons whose wavelengths λ are much larger than r. By exploiting these three theoretical tools (perturbation theory, the relativistic expansion, and the multipole expansion), we hope to understand charmonium physics in sufficient detail to make it a useful probe of heavy-ion collisions.

3. CHARMONIUM PRODUCTION

We now turn to the problem of charmonium production, where we will focus on the production of the J/ψ in hadron collisions. We can think of the production of this state as proceeding through two steps. The first step is the creation of a $c\bar{c}$ pair with small relative momentum. If their relative momentum is much larger than $m_c v$, they will be unlikely to bind and will instead fly apart and form D mesons. The creation of the $c\bar{c}$ pair must involve momenta on the order of m_c or larger, because the momenta of the incoming particles provide the rest energy of the c and \bar{c} . The second step in the production of the ψ is the binding of the $c\bar{c}$ pair. This step necessarily involves momenta of order $m_c v$ or smaller, because gluons whose wavelengths are comparable to the size of the bound state play a large role in the binding.

We first consider the creation of the $c\bar{c}$ pair. One way to produce them is through collisions between partons from the colliding hadrons. The simplest such processes are $gg \to c\bar{c}$ and $q\bar{q} \to c\bar{c}$. Parton collision processes such as these always involve virtual particles that are off their mass-shells by amounts of order m_c . The cross section for creating the $c\bar{c}$ pair can therefore be calculated as a perturbation series in $\alpha_s(m_c)$. One can also show that the $c\bar{c}$ pair from such a process is produced with separation of order $1/m_c$ or smaller, which is much smaller than the size of a quarkonium state. Thus, when the $c\bar{c}$ pair is produced, it is essentially pointlike on the scale of the charmonium wavefunction.

There are also production mechanisms that produce $c\bar{c}$ pairs that are not pointlike. For example, a gluon from one hadron can undergo a quantum fluctuation into a virtual $c\bar{c}$ pair. This pair can make a transition into a real $c\bar{c}$ pair by exchanging soft gluons with the other hadron. The separation of the resulting $c\bar{c}$ pair is comparable to the wavelength of the exchanged gluon. Such processes are particularly important in forward or diffractive

production. Fortunately, one can minimize the complications from such processes by going to a kinematic region where they are suppressed, such as transverse momentum much greater than Λ_{QCD} . We will therefore restrict our attention from this point on to parton collision processes that produce pointlike $c\bar{c}$ pairs.

We now consider the binding of the $c\bar{c}$ pair to form the ψ . Assuming that the $c\bar{c}$ pair is produced through parton collisions, all we need to know is the probability for a pointlike $c\bar{c}$ pair to bind to form a ψ . The inclusive differential cross section can be expressed in a form in which short-distance and long-distance contributions have been factored:

$$d\sigma(\psi + X) = \sum_{n} d\widehat{\sigma} \left((c\overline{c})_n + X \right) \langle \mathcal{O}_n^{\psi} \rangle. \tag{1}$$

The sum over n includes all possible color and angular momentum states of the $c\bar{c}$ pair. The short-distance factor is the cross section $d\hat{\sigma}$ for creating a $c\bar{c}$ pair in the state n, which can be calculated as a perturbation expansion in $\alpha_s(m_c)$. The long-distance factor $\langle \mathcal{O}_n^{\psi} \rangle$ gives the probability for a pointlike $c\bar{c}$ pair in the state n to bind to form the ψ . The matrix elements $\langle \mathcal{O}_n^{\psi} \rangle$ contain all the nonperturbative information that is required to calculate the inclusive cross section.

Until the last few years, most of the predictions for charmonium production in high energy collisions were carried out using the color-singlet model [2]. This model is a very simple prescription for the probability for a pointlike $c\bar{c}$ pair to form a charmonium meson. The factor $\langle \mathcal{O}_n^{\psi} \rangle$ is assumed to be nonzero only if the state n is a color singlet with the same angular-momentum quantum numbers as the dominant Fock state of the meson, which is 3S_1 in the case of the ψ . The probability for a pointlike $c\bar{c}$ pair in the color-singlet 3S_1 state to form the ψ is the same as the probability for the ψ to consist of a pointlike $c\bar{c}$ pair. It is proportional to $|R(0)|^2$, where R(0) is the radial wavefunction at the origin. The color-singlet model is very predictive. It gives predictions for ψ production in all high energy processes in terms of the single nonperturbative factor $|R(0)|^2$. Moreover, the factor $|R(0)|^2$ can be measured experimentally by the decay rate for $\psi \to e^+e^-$. Thus the predictions of the color-singlet model are absolutely normalized.

We now consider the production of prompt charmonium in $p\bar{p}$ collisions. "Prompt" means that the charmonium is produced by QCD interactions rather than by weak decays of hadrons containing bottom quarks. The cross section for prompt ψ production at Fermilab's Tevatron has been measured by the CDF collaboration for transverse momenta in the range 5 GeV $< p_T < 20$ GeV [3]. The background from decays of bottom hadrons were removed using a silicon vertex detector. The contribution from the radiative decays $\chi_{cJ} \to \psi \gamma$ was measured and subtracted. The data is shown in Figure 1. The dotted line gives the prediction of the color-singlet model at large p_T . (The dashed line gives another contribution in the color-singlet model that becomes important at lower p_T .) The result is rather surprising. The measured cross section was found to be about a factor of 30 larger than predicted by the color-singlet model.

This enormous discrepancy between theory and experiment is very surprising, because the data extends out to large p_T where the theoretical analysis is particularly clean. There are factorization theorems of perturbative QCD that guarantee that the inclusive differential cross section for the production of a hadron at large p_T must be dominated by fragmentation, the formation of that hadron in the jet initiated by a parton with large transverse momentum. In the case of charmonium, large transverse momentum means

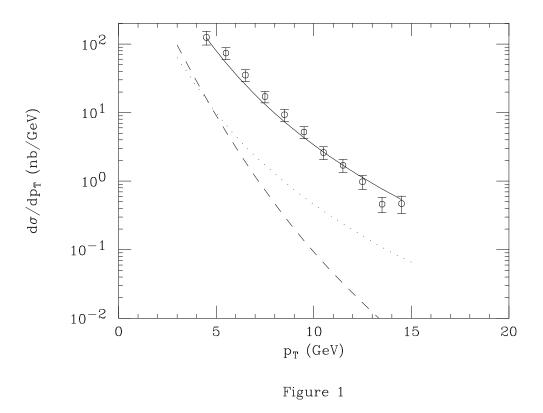


Figure 1. CDF data on the differential cross section for prompt ψ 's that do not come from χ 's as a function of p_T . The dashed and dotted lines are two contributions predicted by the color-singlet model, while the solid line is the prediction of the color-octet mechanism with the normalization adjusted to fit the data.

 $p_T \gg m_c$ and the most important partons are gluons. The inclusive differential cross section can therefore be written in the form

$$d\sigma(\psi(P) + X) = \int_0^1 dz \, d\widehat{\sigma} \left(g(P/z) + X \right) D_{g \to \psi}(z), \tag{2}$$

where $d\hat{\sigma}$ is the differential cross section for producing a gluon with momentum P/z and $D_{g\to\psi}(z)$ is the gluon fragmentation function, which gives the probability that the jet initiated by the gluon will include a ψ carrying a fraction z of the gluon momentum. All of the dependence on p_T resides in the factor $d\hat{\sigma}$, which can be calculated using perturbation theory in $\alpha_s(p_T)$. At leading order, it comes from the parton process $gg \to gg$, which has a cross section of order $\alpha_s^2(p_T)$. All effects from momentum scales of order m_c or smaller are factored into $D_{g\to\psi}(z)$. In particular, the nonperturbative effects associated with the binding of the $c\bar{c}$ pair are all included in the fragmentation function. The color-singlet model gives a definite prediction for the fragmentation function in (2). The leading contribution comes from the parton process $g \to c\bar{c}gg$. The resulting fragmentation function is proportional to $\alpha_s^3(m_c)|R(0)|^2/m_c^3$. The dotted curve in Figure 1 is the prediction obtained using this fragmentation function. Gluon fragmentation gives the dominant contribution in the color-singlet model for p_T greater than about 5 GeV. However, the predicted cross section falls about a factor of 30 below the CDF data.

How can we make sense of this enormous discrepancy between theory and experiment? The factorization theorems summarized by (2) are on a very firm foundation. The cross section must certainly have this form at the largest values of p_T that are available. If the prediction fails to agree with the data, the problem must lie in our assumption about the fragmentation function $D_{g\to\psi}(z)$. The color-singlet model must drastically underestimate the probability for a gluon to fragment into charmonium.

4. NRQCD FACTORIZATION APPROACH

How can the probability for the gluon to fragment into charmonium be so much larger than predicted by the color-singlet model? There is a simple and natural explanation that is based on a new approach to charmonium production developed recently by Bodwin, Braaten, and Lepage [4]. This approach is called the NRQCD factorization formalism, because it makes use of an effective field theory called nonrelativistic QCD. The basic idea is to exploit the relativistic expansion for the long-distance factors $\langle \mathcal{O}_n^{\psi} \rangle$ in the factorization formula (1). The factor $\langle \mathcal{O}_n^{\psi} \rangle$ gives the probability for a pointlike $c\bar{c}$ pair in the color and angular-momentum state labelled by n to bind to form a ψ . It can be expressed as a well-defined NRQCD matrix elements that scales in a definite way with v, the typical relative velocity of the c and \bar{c} . The leading matrix element in v is proportional to $|R(0)|^2$ and scales like v^3 . The corresponding term in the factorization formula (1) is the cross section of the color-singlet model. All other terms have matrix elements that are suppressed by powers of v^2 . Thus, if the parton cross sections $d\hat{\sigma}((c\bar{c})_n + X)$ were all comparable, the color-singlet model would indeed give the dominant term in the cross section

However, the parton cross sections $d\hat{\sigma}((c\bar{c})_n + X)$ can vary widely in magnitude. They can be suppressed not only by powers of $\alpha_s(m_c)$ or $\alpha_s(p_T)$, but also by powers of kinematical factors like m_c/p_T . If the color-singlet model term is suppressed by such a factor, then

other terms may be important in spite of their suppression by powers of v^2 . After $|R(0)|^2$, the next most important matrix elements are $\langle \mathcal{O}_8^{\psi}(^3S_1) \rangle$, $\langle \mathcal{O}_8^{\psi}(^1S_0) \rangle$, and $\langle \mathcal{O}_8^{\psi}(^3P_0) \rangle$, all of which are suppressed by v^4 . They give the probabilities for forming a ψ from pointlike color-octet $c\bar{c}$ pairs in spin-triplet S-wave, spin-singlet S-wave, and spin-triplet P-wave states, respectively. These matrix elements are nonperturbative quantities that cannot be related in any simple way to the $c\bar{c}$ wavefunction. We have no effective means of calculating them from first principles. These matrix elements must therefore be treated as phenomenological parameters to be determined from experimental data.

Now consider the fragmentation function $D_{g\to\psi}(z)$ that describes the formation of a ψ from a high- p_T gluon. The color-singlet model term in the fragmentation function has a short-distance factor of order $\alpha_s^3(m_c)$ and a long-distance factor that scales like v^3 . All other terms have long-distance factors that are suppressed by powers of v^2 , but there are some for which the short-distance factor is lower order in α_s . In particular, the term that corresponds to the parton process $g\to c\bar{c}$ has a short-distance factor that is only of order α_s . The matrix element in this term is $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$, which gives the probability of forming a ψ from a pointlike $c\bar{c}$ pair in a color-octet 3S_1 state. The fragmentation function at leading order in α_s is

$$D_{g\to\psi}(z) = \frac{\pi\alpha_s(m_c)}{24m_c^3}\delta(1-z)\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle. \tag{3}$$

This term in the fragmentation function is of order $\alpha_s v^7$, compared to $\alpha_s^3 v^3$ for the color-singlet term. It is not immediately obvious which of these two terms is more important, but experience tells us that suppression by a power of α_s tends to be more effective than suppression by a power of v^2 . Thus it is not unreasonable for this color-octet term to dominate.

If the color-octet term is sufficiently large, it could provide an explanation for the CDF data on prompt charmonium production [5]. The color-octet term in the cross section at large p_T is obtained by inserting the fragmentation function (3) into (2). The resulting prediction for the differential cross section as a function of p_T is shown as a solid line in Figure 1. Its shape fits the data quite well. The matrix element $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$ can be adjusted so that the curve also agrees with the data in normalization. The value of the matrix element that is required is $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle = 0.014~\text{GeV}^3$. Now for this explanation of the data to be viable, the matrix element must satisfy an important consistency check. It must be small enough to be consistent with suppression by v^4 relative to the corresponding color-singlet factor $|R(0)|^2$. Translating both quantities into the probability density for a point-like $c\bar{c}$ pair to bind to form a ψ , we get $21/\text{fm}^3$ for the color-singlet 3S_1 state and $1.8/\text{fm}^3$ for the color-octet 3S_1 state. The latter value is perfectly consistent with suppression by a factor of v^4 .

We have shown that a color-octet term in the gluon fragmentation function provides a plausible explanation for the CDF data on prompt ψ production. To make this explanation convincing, we have to show that this color-octet production mechanism explains other aspects of charmonium production. There are two particularly dramatic predictions that should be tested experimentally in the near future. The first prediction is that at the Tevatron, ψ' 's at large p_T should be almost completely transversely polarized [6]. The ψ' essentially inherits the transverse polarization of the fragmenting gluon. There is an effort underway at CDF to measure this polarization. The second prediction is that prompt ψ 's should be observable in Z^0 decay at LEP [7]. The color-singlet model predicts a rate for prompt ψ production that is too small to be observed. Using the value of $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$ obtained by fitting the CDF data, we find that the color-octet mechanism increases the production rate by almost an order of magnitude. The predicted branching fraction for $Z^0 \to \psi + X$ is 1.4×10^{-4} , which is large enough that it should be observable. Preliminary measurements of prompt ψ production from some of the LEP detectors seem to indicate that the branching fraction does indeed have the magnitude predicted by the color-octet production mechanism.

5. CONCLUSIONS

If we understand charmonum production in simple high energy collisions, it should be possible to predict the production rate in heavy-ion collisions. A new framework has been developed for describing charmonium production in those kinematic regions where it is dominated by parton collisions. The cross section is factored into parton cross sections that can be computed using perturbation theory and well-defined matrix elements that give the probability for forming the bound state. The matrix elements with the greatest phenomenological importance include $|R(0)|^2$, $\langle \mathcal{O}_8(^3S_1)\rangle$, $\langle \mathcal{O}_8(^1S_0)\rangle$, and $\langle \mathcal{O}_8(^3P_0)\rangle$ for ψ and ψ' and $|R'(0)|^2$ and $\langle \mathcal{O}_8(^3S_1)\rangle$ for χ_{cJ} . It should be possible to determine these factors with reasonable precision by carrying out a thorough analysis of all the available data on charmonium production, including fixed target data from pN, πN and eN collisions as well as data from the high energy $p\bar{p}$, e^+e^- , and ep colliders. The resulting matrix elements can then be used as inputs for predictions of charmonium production in heavy-ion collisions.

In conclusion, new data from high energy colliders has been driving progress in heavy quarkonium physics. There are powerful theoretical tools that can be used to study heavy quarkonium. The developments described above have exploited two of these tools: perturbation theory and the relativistic expansion. The multipole expansion is another powerful tool that has important implications for heavy quarkonium production [8]. By exploiting these tools, it should be possible to understand heavy quarkonium in sufficient detail to allow it to be used as a probe in heavy-ion collisions.

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